accepted for publication in *Physical Review E* 2002

Scaling behavior of the conserved transfer threshold process

S. Lübeck*

Weizmann Institute of Science, Department of Physics of Complex Systems, 76100 Rehovot, Israel,
Theoretische Tieftemperaturphysik, Gerhard-Mercator-Universität, 47048 Duisburg, Germany

(Dated: June 20, 2002)

We analyze numerically the critical behavior of an absorbing phase transition in the conserved transfer threshold process. We determined the steady state scaling behavior of the order parameter as a function of both, the control parameter and an external field, conjugated to the order parameter. The external field is realized as a spontaneous creation of active particles which drives the system away from criticality. The obtained results yields that the conserved transfers threshold process belongs to the universality class of absorbing phase transitions in a conserved field.

PACS numbers: 05.70.Ln, 05.50.+q, 05.65.+b

I. INTRODUCTION

The scaling behavior of directed percolation is recognized as the paradigm of the critical behavior of several non-equilibrium systems which exhibits a continuous phase transition from an active state to an absorbing non-active state (see for instance [1]). The widespread occurrence of such systems in physics, biology, as well as catalytic chemical reactions is reflected by the well known universality hypothesis of Janssen and Grassberger that models which exhibit a continuous phase transition to a single absorbing state generally belong to the universality class of directed percolation [2, 3]. Introducing additional symmetries the critical behavior differs from directed percolation. In particular particle conservation leads to the different universality class of absorbing phase transitions with a conserved field as pointed out in [4]. In that work the authors introduced two models, the conserved lattice gas (CLG) and the conserved threshold transfer process (CTTP). The latter one is a conserved modification of the threshold transfer process introduced in [5]. Both models display a continuous phase transition from an active to an inactive phase and are believed to belong to the same universality class [4]. The steady-state scaling behavior of the CLG model was investigated recently. The order parameter and its fluctuations were numerically examined in [6]. The scaling behavior in an external field conjugated to the order parameter was considered in [7]. Furthermore a modified CLG model was introduced which allows to determine analytically the steady-state mean-field scaling behavior of the universality class [8, 9].

On the other hand the scaling behavior of the CTTP was investigated in low dimensional ($D = 1, 2$) systems only [4, 10] and no external field was applied. Therefore we consider in this work the CTTP with and without an external field in various dimensions ($D = 1, 2, 3, 4, 5, 6$) and determine a set of critical exponents. All obtained results coincides with those of the CLG model, strongly supporting the universality hypothesis of [4].

*Electronic address: sven@thp.uni.duisburg.de

II. MODEL AND SCALING BEHAVIOR

In this work we consider the CTTP on simple cubic lattices of linear size L in various dimensions with N particles. The lattice sites may be empty, occupied by one particle, or occupied by two particles. Empty and single occupied sites are considered as non-active whereas double occupied lattice sites are considered as active. In the latter case one tries to transfer both particles of each active site to randomly chosen empty or single occupied nearest neighbor sites. If no active sites exist the system is trapped forever in a certain configuration, a so-called absorbing state.

In the following we denote the densities of active sites with $\rho_{\rm a}$ and the density of particles on the lattice as $\rho = N/L^D$, which is considered as the control parameter of the absorbing phase transition. The density of active sites $\rho_{\rm a}$ is the order parameter of the absorbing phase transition, i.e., it vanishes at the critical density $\rho_{\rm c}$ according to

$$\rho_{\rm a} \sim \delta\rho^{\beta}, \qquad (1)$$

with the reduced control parameter $\delta\rho = \rho/\rho_{\rm c} - 1$ and the order parameter exponent β.

Similar to equilibrium phase transitions it is possible in the case of absorbing phase transitions to apply an external field h which is conjugated to the order parameter (see for instance [1]). As usually for continuous phase transitions the conjugated field has to destroy the disordered phase and the associated linear response function $\partial\rho_{\rm a}/\partial h$ has to diverge at the critical point ($\delta\rho = 0$, $h = 0$). In the case of an absorbing phase transition the external field acts as a spontaneous creation of active particles, i.e., the external field destroys the absorbing state and thus the phase transition itself. But considering absorbing phase transitions with particle conversation one has to take care that the external field does not change the particle number. A possible realization of the external field was developed in [7] where the external field triggers movements of inactive particles which may be activated in this way. The external field h is another relevant scaling field and for sufficiently small values of h

the order parameter scales as

$$\rho_{\rm a}(\delta\rho, h) \;\sim\; \lambda\, \tilde{r}(\delta\rho\, \lambda^{-1/\beta}, h\, \lambda^{-\sigma/\beta}) \tag{2}$$

with the critical field exponent σ and the scaling function $\tilde{r}$. Choosing $\delta\rho\, \lambda^{-1/\beta} = 1$ one recovers Eq. (1) whereas $h\, \lambda^{-\sigma/\beta} = 1$ leads at the critical density to

$$\rho_{\rm a} \sim h^{\beta/\sigma}. \tag{3}$$

In our simulations we start with randomly distributed particles. All active sites are listed and this list is updated in a randomly chosen sequence. In the case that an external field is applied the active particle creation is performed after each update step in order to mimic the external field. After a certain relaxation time the system reaches a steady state where the density of active sites at update step t fluctuates around the average value $\langle\rho_{\rm a}(\delta\rho, h, t)\rangle$ which is interpreted as the order parameter $\rho_{\rm a}(\delta\rho, h)$ (see for instance Figs. 1 of [6, 7]).

Additionally to the order parameter we consider its fluctuations

$$\Delta\rho_{\rm a}(\delta\rho, h) \;=\; L^D \,\left[\langle\rho_{\rm a}(\delta\rho, h, t)^2\rangle \,-\, \langle\rho_{\rm a}(\delta\rho, h, t)\rangle^2\right]. \tag{4}$$

Approaching the transition point the fluctuations diverge for zero-field according to

$$\Delta\rho_{\rm a}(\delta\rho, h=0) \;\sim\; \delta\rho^{-\gamma'}. \tag{5}$$

The fluctuation exponent γ' fulfills the scaling relation [11]

$$\gamma' \;=\; \nu_{\perp} D \,-\, 2\,\beta, \tag{6}$$

where the exponent $\nu_{\perp}$ describes how the spatial correlation length diverges at the transition point. In the critical regime we assume that the fluctuations obey the scaling ansatz

$$\Delta\rho_{\rm a}(\delta\rho, h) \;=\; \lambda^{\gamma'}\, \tilde{d}(\delta\rho\, \lambda, h\, \lambda^{\sigma}). \tag{7}$$

Setting $\delta\rho\, \lambda = 1$ one recovers Eq. (5) for $h=0$.

Analogous to equilibrium phase transitions the susceptibility is defined as the derivative of the order parameter with respect to the conjugated field

$$\begin{aligned}\chi(\delta\rho, h) \;&=\; \frac{\partial}{\partial h}\, \rho_{\rm a}(\delta\rho, h) \\ &=\; \lambda^{1-\sigma/\beta}\, \tilde{c}(\delta\rho\, \lambda^{-1/\beta}, h\, \lambda^{-\sigma/\beta}).\end{aligned} \tag{8}$$

Setting $\delta\rho\, \lambda^{-1/\beta} = 1$ one gets that the susceptibility diverges for zero-field as required according to

$$\left.\frac{\partial\rho_{\rm a}}{\partial h}\right|_{h\to 0} \;\sim\; \delta\rho^{-\gamma}. \tag{9}$$

Furthermore, one yields the scaling relation

$$\gamma \;=\; \sigma \,-\, \beta \tag{10}$$

which corresponds to the well known Widom equation of equilibrium phase transitions. Using this scaling relation one can calculate the value of the susceptibility exponent γ from the obtained values of β and σ. Notice that in contrast to the scaling behavior of equilibrium phase transitions the non-equilibrium absorbing phase transition is characterized by $\gamma \neq \gamma'$.

III. BELOW THE UPPER CRITICAL DIMENSION

At the beginning of our analysis we consider the scaling behavior of the order parameter for $D = 1, 2, 3$. System sizes up to $L = 131072$ for $D = 1$, $L = 2048$ for $D = 2$, and $L = 256$ for ($D = 3$) are considered. In each cases we start the simulation with randomly distributed particles. After a certain transient regime the system reaches a steady state where the density of active particles fluctuates around an average value which is interpreted as the order parameter. In the steady state up to $2\,10^8$ update steps for $D = 1$ and $2\,10^6$ for $D = 2, 3$ are performed to measure the average density of active sites. For zero-field this procedure is repeated for at least 10 different initial configurations in order to get an accurate estimation of the order parameter close to the critical point ($\rho = \rho_{\rm c}$, $h = 0$).

In Fig. 1 we present the data of the one-dimensional order parameter at zero-field. Approaching the transition point the corresponding correlation length increases and the system tends to the absorbing state if the correlation length is of the order of the system size. Instead of a finite-size scaling analysis (see for instance [4, 10, 12]) we take care of these finite-size effects in the way that

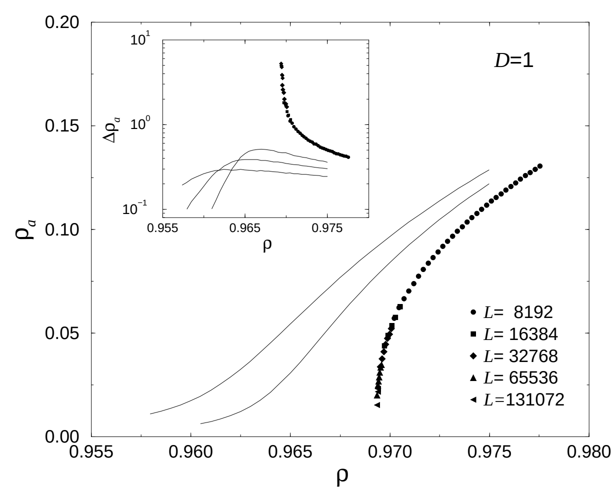

FIG. 1: The order parameter $\rho_{\rm a}$ as a function of the particle density for zero-field (symbols) and for $h = 0.0001$ and $h = 0.00005$ (lines). The inset displays the order parameter fluctuations $\Delta\rho_{\rm a}$ for zero field (symbols) and for $h = 0.0002$, $h = 0.0001$ and $h = 0.00005$ (lines).

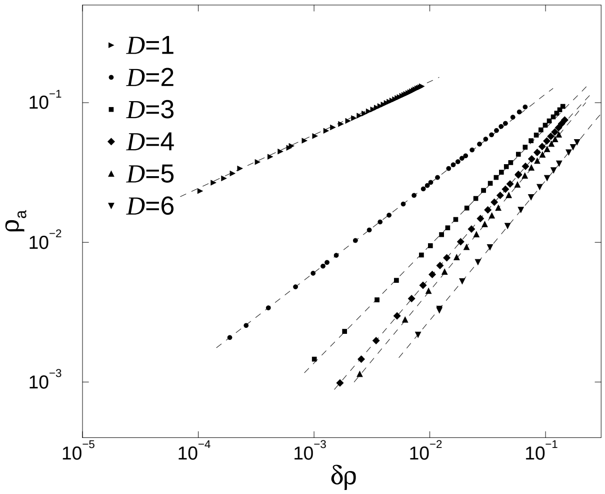

FIG. 2: The order parameter ρ_a as a function of the reduced particle density $\delta\rho$ at zero-field for various dimensions D. The dashed line corresponds to a power-law behavior according to Eq. (1) for $D \neq D_c$. For $D = 6$ the data are shifted horizontally by a factor 1.5 in order to avoid an overlap. In the case of the four-dimensional model the dashed line corresponds to Eq. (14) with $B = 0.15$.

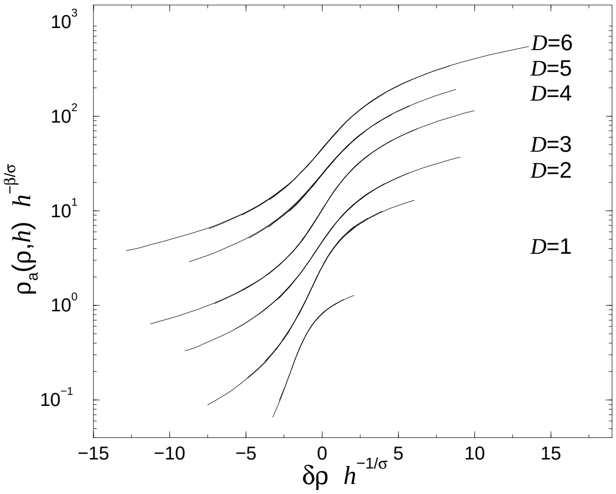

FIG. 4: The scaling plot of the order parameter fluctuations $\Delta\rho_a$ for various dimensions. For $D > 1$ the curves are shifted vertically in order to avoid overlaps. In the case of the four-dimensional model $\rho_a\, h^{-1/2}\, |\ln h|^{-\Sigma}$ is plotted vs. $\delta\rho\, h^{-1/2} |\ln h|^{b-s/2}$ with $\Sigma = 0.28$ and $b - s/2 = -0.12$ (see text).

we increase the system size before these finite-size effects occur and use only data from simulations that have not reached the absorbing state.

Decreasing the particle density the order parameter decreases and vanishes at the transition point. To determine the critical indices one varies the critical density ρ_c until one obtains asymptotically a straight line in a log-log plot. The exponent is then obtained by a regression analysis. The values of the order parameter as a function of the reduced particle density $\delta\rho$ are plotted in Fig. 2. In all cases the asymptotic behavior ($\delta\rho \to 0$) of the order parameter obeys Eq. (1). For $D = 1$ we get $\rho_c = 0.96929 \pm 0.00003$ and $\beta = 0.382 \pm 0.019$. The latter value is smaller than the value $\beta = 0.412$ estimated from significantly smaller system sizes ($L \leq 5000$) [10]. Furthermore our value differs from $\beta = 0.42 \pm 0.02$ obtained from simulations of the one-dimensional fix-energy Manna sandpile model [12] that is expected to belong to the same universality class.

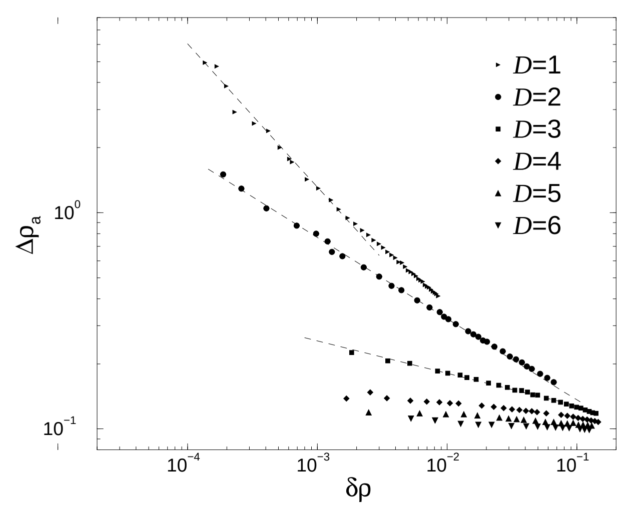

FIG. 3: The order parameter fluctuations $\Delta\rho_a$ as a function of the reduced particle density $\delta\rho$ at zero-field for various dimensions D. The dashed line corresponds to a power-law divergence [Eq. (5)]. For $D \geq D_c$ the fluctuations are maximal at the transition point but finite.

In the two dimensional case we obtain $\rho_c = 0.69392 \pm 0.00001$ and $\beta = 0.639 \pm 0.009$. Again the order parameter exponent differs slightly from the previously reported result $\beta = 0.656$ obtained from simulations of small lattice sizes ($L \leq 160$) [10]. But our value agrees with the estimate of the corresponding two-dimensional Manna sandpile model $\beta = 0.64 \pm 0.01$ [13].

The estimates of the three dimensional model are $\rho_c = 0.60489 \pm 0.00002$ and $\beta = 0.840 \pm 0.012$. All obtained critical exponents are listed in Table I.

In Fig. 3 we present the order parameter fluctuations as a function of the control parameter at zero field. We observe for $D < D_c$ a power-law behavior according to Eq. (5). Using a regression analysis we get the estimates $\gamma' = 0.662 \pm 0.071$ for $D = 1$, $\gamma' = 0.381 \pm 0.013$ for $D = 2$, and $\gamma' = 0.187 \pm 0.030$ for $D = 3$.

In the following we analyze the order parameter as a function of the control parameter $\delta\rho$ for different fields from $h = 10^{-5}$ up to $2\,10^{-4}$. The applied field results in a smoothing of the zero-field curve, i.e., the order parameter increases smoothly with the control parameter for $h > 0$ (see Fig. 1). According to the scaling ansatz of

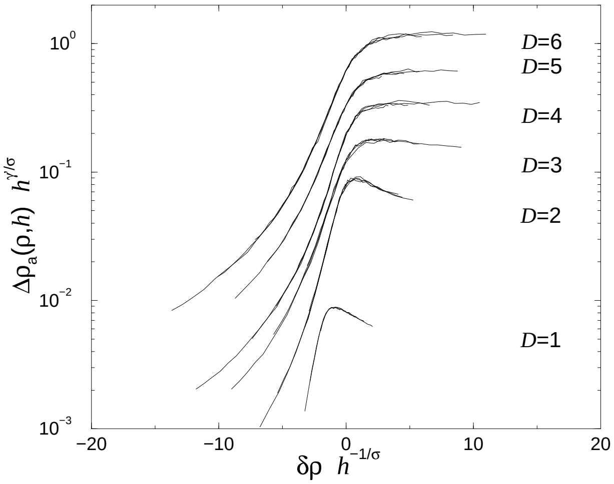

FIG. 5: The scaling plots of the order parameter fluctuations $\Delta\rho_{\rm a}$ for various dimensions. For $D>1$ the curves are shifted vertically in order to avoid overlaps. The fluctuations diverges at the critical point for $D<4$ whereas a jump of the fluctuations is observed in higher dimensions at zero-field. In the in case of the four-dimensional model $\Delta\rho_{\rm a}$ is plotted vs. $\delta\rho\, h^{-1/2}\,|\ln h|^{-\eta}$ with $\eta = 0.1$.

the order parameter [Eq. (2)] we choose $h\,\lambda^{-\sigma/\beta}=1$ and get the scaling form

$$\rho_{\rm a}(\delta\rho, h) \ = \ h^{\beta/\sigma}\ \tilde{r}(\delta\rho\ h^{-1/\sigma}, 1). \tag{11}$$

Thus one varies the exponent σ until the curves for different values of the driving field have to collapse onto the scaling function $\tilde{r}$ if one plots $\rho_{\rm a}\,h^{-\beta/\sigma}$ as a function of $\delta\rho\, h^{-1/\sigma}$. Convincing results are obtained for $\sigma = 1.770 \pm 0.058$ ($D=1$), $\sigma = 2.229 \pm 0.032$ ($D=2$), as well as $\sigma = 2.069 \pm 0.043$ ($D=3$) and the corresponding scaling plots are shown in Fig. 4.

Next we consider the scaling behavior of the order parameter fluctuations $\Delta\rho_{\rm a}$. The fluctuation data for $D=1$ are shown for different values of the external field in the inset of Fig. 1. For finite fields the fluctuations display a peak. Approaching the transition point ($h\to 0$) this peak becomes a divergence signalling the critical point. In order to analyze the scaling behavior of the fluctuations we use the scaling ansatz Eq. (7) and set $h\,\lambda^{\sigma}=1$

$$\Delta\rho_{\rm a}(\delta\rho, h) \ = \ h^{-\gamma'/\sigma}\ \tilde{d}(\delta\rho\ h^{-1/\sigma}, 1). \tag{12}$$

Using the above determined values of $\rho_{\rm c}$, σ and γ' we get good data collapses confirming the accuracy of our analysis. (see Fig. 5)

Furthermore we determine the susceptibility exponent γ. Using the scaling relation Eq. (10) one gets the estimates of the susceptibility exponents $\gamma = 1.388 \pm 0.063$ ($D=1$), $\gamma = 1.590 \pm 0.033$ ($D=2$), and $\gamma = 1.229 \pm 0.045$ ($D=3$).

IV. AT THE UPPER CRITICAL DIMENSION

In the case of the four dimensional model we considered system sizes from $L=8$ up to $L=64$. At least 10^6 update steps were used to reach the steady state close to the transition point and $2\,10^6$ update steps were performed to determine the order parameter and its fluctuations.

At the upper critical dimension $D_{\rm c}=4$ the scaling behavior of the CTTP is affected by logarithmic corrections similar to the CLG model [6, 7]. As argued in [7] the order parameter obeys in leading order the scaling ansatz

$$\rho_{\rm a}(\delta\rho, h) \ = \ \lambda\,|\ln\lambda|^{l}\ \tilde{r}(\delta\rho\,\lambda^{-1/\beta}|\ln\lambda|^{b}, h\,\lambda^{-\sigma/\beta}|\ln\lambda|^{s}), \tag{13}$$

where the exponents β and σ are given by the corresponding mean-field values $\beta=1$ and $\sigma=2$ [9], respectively. Thus, for zero field the asymptotic scaling behavior of the order parameter obeys

$$\rho_{\rm a}(\delta\rho, h=0) \ \sim \ \delta\rho\,|\ln\delta\rho|^{\rm B} \tag{14}$$

with $\mathrm{B}=b+l$. In our analysis we plot $\rho_{\rm a}/\delta\rho$ as a function of $|\ln h|^{\rm B}$ and vary the exponent B as well as the critical density $\rho_{\rm c}$ until one gets asymptotically a straight line. The best result is obtained for $\mathrm{B}=0.15$, $\rho_{\rm c} = 0.56705 \pm 0.00003$ and the corresponding plot is shown in Fig. 6. This value of B differs from the corresponding value of the CLG model $\mathrm{B}=0.24$ [7].

Similar to the lower dimensions we consider the scaling behavior of the order parameter as a function of the control parameter for different external fields. Choosing $h\,\lambda^{-\sigma/\beta}|\ln\lambda|^{s}=1$ the scaling ansatz [Eq. (13)] yields in leading order

$$\rho_{\rm a}(\delta\rho, h) \ = \ h^{1/2}\,|\ln h|^{\Sigma}\ \tilde{r}(x, 1), \tag{15}$$

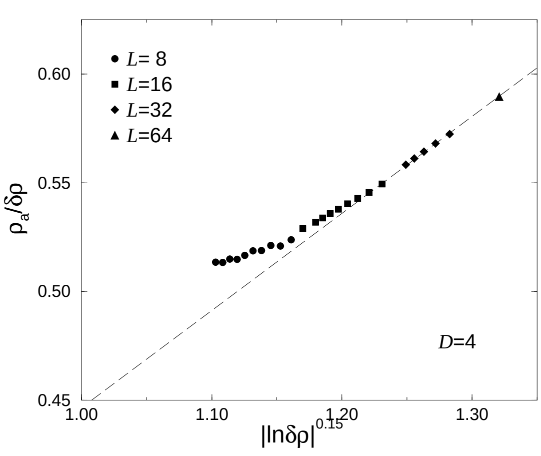

FIG. 6: The density of active sites at the upper critical dimension $D_{\rm c}=4$. The data are rescaled according to Eq. (14). The assumed asymptotic scaling behavior (dashed line) is obtained for $\mathrm{B}=0.15$.

where the scaling argument x is given in leading order by

$$x \;=\; \delta\rho \, h^{-1/2} \, |\ln h|^{b-s/2} \tag{16}$$

with $\Sigma = s/2 + l$. Varying the logarithmic correction exponents one gets for $\Sigma = 0.28$ and $b - s/2 = -0.12$ a convincing data-collapse, which is shown in Fig. 4. Using the values $\Sigma = l + s/2 = 0.28$ and $b - s/2 = -0.12$ we get the estimation $\mathrm{B} = b + l = 0.16$ which agrees with $\mathrm{B} = 0.15$ obtained from numerical simulations in zero-field. On the other hand this values differs from the corresponding estimations of the CLG model $\mathrm{B} = 0.24$, $\Sigma = 0.45$, and $b - s/2 = -0.17$ [7].

Furthermore we consider how the logarithmic corrections affect the scaling behavior of the fluctuations at the upper critical dimension. As pointed out in [7] the order parameter fluctuations are expected to obey the scaling ansatz

$$\Delta\rho_{\mathrm{a}}(\delta\rho, h) \;=\; \tilde{d}(\delta\rho \, h^{-1/2} \, |\ln h|^{-\eta}, 1). \tag{17}$$

A good data collapse is observed for $\eta = 0.10$ (see Fig. 5) which differs again from the corresponding value of the CLG model $\eta = 0.39$ [7].

V. ABOVE THE UPPER CRITICAL DIMENSION

A modified version of the CTTP with random neighbor hopping was recently introduced in [9]. There, unrestricted particle hopping breaks long range correlations and the scaling behavior is characterized by the mean-field values $\rho_{\mathrm{c}} = 1/2$, $\beta = 1$, and $\sigma = 2$ which are calculated analytically.

In our simulations of the five dimensional model we considered system sizes from $L = 8$ up to $L = 32$ whereas system sizes from $L = 4$ up to $L = 16$ are used for $D = 6$. At least $2\,10^6$ update steps were used to reach the steady state and $2\,10^6$ update steps were performed to determine the order parameter and its fluctuations. The values of the order parameter are plotted in Fig. 2 and the obtained critical densities are $\rho_{\mathrm{c}} = 0.54864 \pm 0.00005$ for $D = 5$ and $\rho_{\mathrm{c}} = 0.53816 \pm 0.00007$ for $D = 6$, respectively. In both dimensions the asymptotic scaling behavior of the order parameter is in agreement with the mean-field behavior $\beta = 1$.

The fluctuations of the order parameter $\Delta\rho_{\mathrm{a}}$ are plotted in Fig. 3. Analogous to the CLG model the fluctuations are characterized by a jump at the transition point corresponding to $\gamma' = 0$ [6].

Above the critical dimension, i.e. $D \geq 5$, the scaling behavior of the CTTP is expected to obey again the scaling ansatzes Eqs. (2,7) where the exponents are given by the mean-field values independently of the particular dimension. The obtained data collapse of the order parameter curves are presented in Fig. 4 and confirm the above scenario.

Furthermore we consider the fluctuations above the upper critical dimension. According to the mean-field value $\gamma' = 0$ [6] we plot $\Delta\rho_{\mathrm{a}}$ as a function of $\delta\rho \, h^{-1/2}$ and the obtained data collapses are shown in Fig. 5.

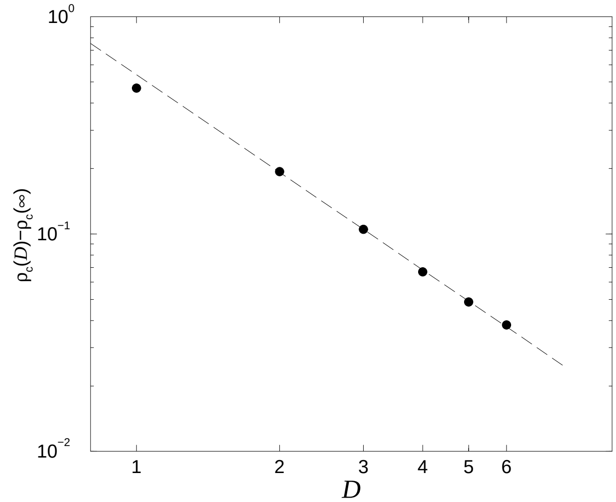

FIG. 7: The critical density $\rho_{\mathrm{c}}(D)$ as a function of the dimension D. The critical density of the mean-field solution is denoted by $\rho_{\mathrm{c}}(\infty) = 1/2$. The dashed line corresponds to a power-law behavior [Eq. (18)] with an exponent 1.48.

Finally we address the question how the critical densities depends on the dimension. As can be seen from Table I the critical density tends with increasing dimension to the mean-field value $\rho_{\mathrm{c}} = 1/2$ [9] that corresponds to an infinite dimension. Our analysis reveals that the critical densities approaches that mean-field value according to

$$\rho_{\mathrm{c}}(D) \;-\; \frac{1}{2} \;\sim\; D^{-\tau} \tag{18}$$

with $\tau = 1.48 \pm 0.05$ (see Fig. 7). This behavior is dif-

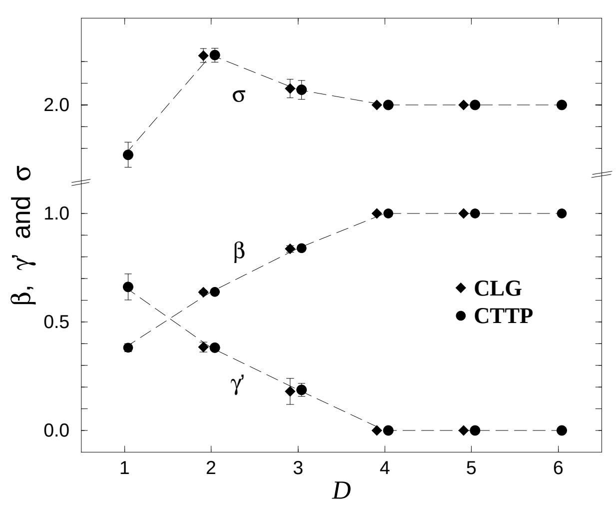

FIG. 8: The critical exponents β, γ', and σ of the CTTP and CLG model (obtained from [6, 7]) for various dimensions. The dashed lines are just to guide the eyes.

ferent from that of CLG models on simple cubic lattices which is characterized by an exponent $\tau = 1$ [8].

VI. CONCLUSIONS

We investigated the steady state scaling behavior of the CTTP model in various dimensions. The order parameter exponent, the fluctuation exponent and the external field exponent are determined and the corresponding values are listed in Table I. For $D = 1$ and $D = 2$ our results of the order parameter exponents differ from previous simulations obtained from significantly smaller system sizes. Our values of the critical exponents β, γ', and σ agree within the error-bars with the corresponding exponents of the CLG model (see Fig. 8), strongly supporting the conjecture [4] that both models belong to the same universality class.

The picture is not so clear at the upper critical dimension $D_c = 4$. Although the exponents are identical the logarithmic correction exponents of the CTTP and CLG model are different. This result is rather surprising since the logarithmic corrections exponents are a characteristic feature of the whole universality class (see for instance [14]). We think that more than statistical uncertainties this result is caused by systematic uncertainties of our analysis. In all cases we focused our attention to the leading order of the scaling behavior. Taking corrections to the leading order into account may result in comparable values of the logarithmic correction exponents. Further investigations are needed to clarify this point.

Acknowledgments

I would like to thank A. Hucht, H.-K. Janssen, and R. D. Willmann for helpful discussions. This work was financially supported by the Minerva Foundation (Max Planck Gesellschaft).

TABLE I: The critical density ρ_c and the critical exponents β, σ, γ' and γ of the CTTP model for various dimensions D. The values of the susceptibility exponent γ are calculated via Eq. (10). The symbol * denotes logarithmic corrections to the power-law behavior.

D	ρ_c	β	σ	γ'	γ
1	0.96929	0.382 ± 0.019	1.770 ± 0.058	0.662 ± 0.071	1.388 ± 0.063
2	0.69392	0.639 ± 0.009	2.229 ± 0.032	0.381 ± 0.013	1.590 ± 0.033
3	0.60489	0.840 ± 0.012	2.069 ± 0.043	0.187 ± 0.030	1.229 ± 0.045
4	0.56705	1^*	2^*	0^*	1^*
5	0.54864	1	2	0	1
6	0.53816	1	2	0	1

[1] H. Hinrichsen, Adv. Phys. **49**, 815 (2000).
[2] H. K. Janssen, Z. Phys. B **42**, 151 (1981).
[3] P. Grassberger, Z. Phys. B **47**, 365 (1982).
[4] M. Rossi, R. Pastor-Satorras, and A. Vespignani, Phys. Rev. Lett. **85**, 1803 (2000).
[5] J. F. F. Mendes, R. Dickman, M. Henkel, and M. C. Marques, J. Phys. A **27**, 3019 (1994).
[6] S. Lübeck, Phys. Rev. E **64**, 016123 (2001).
[7] S. Lübeck, Phys. Rev. E **65**, 046150 (2002).
[8] S. Lübeck and A. Hucht, J. Phys. A **34**, L577 (2001).
[9] S. Lübeck and A. Hucht, J. Phys. A **35**, 4853 (2002).
[10] R. Dickman, T. Tomé, and M. J. de Oliveira, Phys. Rev. E **66**, 016111 (2002).
[11] I. Jensen and R. Dickman, Phys. Rev. E **48**, 1710 (1993).
[12] R. Dickman, M. Alava, M. A. Muñoz, J. Peltola, A. Vespignani, and S. Zapperi, Phys. Rev. E **64**, 056104 (2001).
[13] A. Vespignani, R. Dickman, M. A. Muñoz, and S. Zapperi, Phys. Rev. E **62**, 4564 (2000).
[14] P. Pfeuty and G. Toulouse, *Introduction to the renormalization group and critical phenomena* (John Wiley & Sons, Chichester, 1994).